Digital Control Summary

By: mohammed fathy

D/A Converter $\longrightarrow$ holding circuit.

o/p of the sampler is called star signal or sampled signal.

o/p of sampler of impulse

$$x^*(t) = \sum_{K=0}^{\infty} x(t) \cdot \delta(t-KT) \quad \begin{cases} K \Rightarrow \text{Sampling no.} \\ T \Rightarrow \text{Sampling Period} \end{cases}$$

$$x(z) = \sum_{K=0}^{\infty} x(KT)\, z^{-K}$$

Z-T for some functions

Function	Z.T	Function	Z.T
$u(t)$	$\frac{z}{z-1}$	$\sin(wt)$	$\frac{z\sin(wT)}{z^2-2z\cos(wT)+1}$
e^{at}	$\frac{z}{z-e^{aT}}$	$\cos(wt)$	$\frac{z(z-\cos(wT))}{z^2-2z\cos(wT)+1}$
$\delta(t)$	1	$e^{aT} f(t)$	$F(z)\Big\vert_{z \to z e^{\mp aT}}$
a^t	$\frac{z}{z-a^T}$	$a^t f(t)$	$F(z)\Big\vert_{z = \frac{z}{a^T}}$
t	$\frac{Tz}{(z-1)^2}$		

function	Z.T
$t\,f(t)$	$-Tz\frac{d}{dz}F(z)$
$Z[x(K-n)]$	$z^{-n}X(z) \Rightarrow X(z)=Z[x(K)]$
$Z[x(K+n)]$	$z^{n}X(z)-z^{n}x(0)-z^{n-1}x(1)\ldots\ldots-z\,x(n-1)$

* initial value

$$f(0)=\lim_{t\to 0} f(t)=\lim_{z\to\infty} F(z)$$

* final value (value at $t=\infty$)

$$f(\infty)=\lim f(t)=\lim_{z\to 1}(z-1)\,F(z)=\lim_{z\to 1}(1-z^{-1})F(z)$$

Zero order hold

$$G_h(s)=\frac{1-e^{-Ts}}{s} \xrightarrow{Z.T} 1$$

$$Z[G_h(s).G_1(s)]=Z\left(\frac{1-e^{-Ts}}{s}.G_1(s)\right)=(1-z^{-1}).Z\left[\frac{G_1(s)}{s}\right]$$

→ Pulse T.F

$$\text{Pulse T.F}=\frac{C(z)}{R(z)}$$

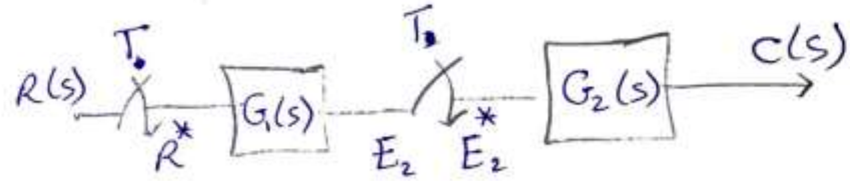
→ we can obtain Pulse T.F from:

a) difference equation like: $y(K-1) + 2y(K) = r(K)$

By: Z.T we get $\frac{Y(z)}{R(z)}$ = Pulse T.F

b) from block diagram → open loop. / → close loop.

① open loop



١) نسمى خرج ودخل ال (sampler) زى $R^* ، R ، E_1 ، E_2^*$

2) نحسب معادلات النظام

$C(s) = G_2(s) \cdot E_2^*$

$E_2 = G_1(s) \cdot R^*$

3) نعمل (staring) للمعادلتين

$C^* = G_2^* \cdot E_2^*$ ، $E_2^* = G_1^* R^*$

← نعوض ونحسب

Pulse T.F $= \frac{C(z)}{R(z)} = \frac{C^*}{R^*}$

← لو طلب (unit step response) وعنده $\frac{C(z)}{R(z)}$

$r(t) = 1 \longrightarrow R(z) = \frac{z}{z-1}$

← بعدين تقوم بحساب $C(z)$ وتجيب منها $c(K)$

بـ $Z^{-1}.T$.

← لو مفيش (Sampler) بين ال (2 blocks) ا نيجي نعمل (staring) هيتم التعامل معاهم كعنصر واحد

- مثلاً: $\overline{G_1 G_2}^*(s) \Leftarrow \overline{G_1 G_2}(z)$

2) <u>closed loop system</u> (to find Pulse T.F)

1) نسمي دخل وخرج ال (Sampler)

2) نحسب معادلة الخرج الرئيسي $C(s)$ بالإضافة لمعادلات ال i/ps الخاصة بال (Sampler)

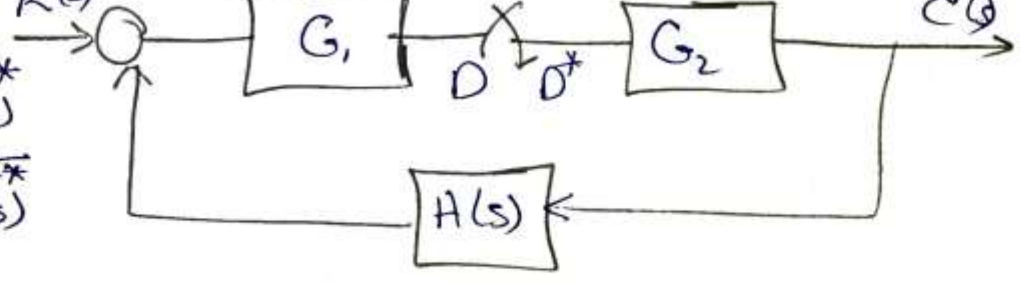
3) قبل عملية (staring) اذا تواجدت تعويضات نقوم بعملها

4) نعمل (staring) للمعادلات.

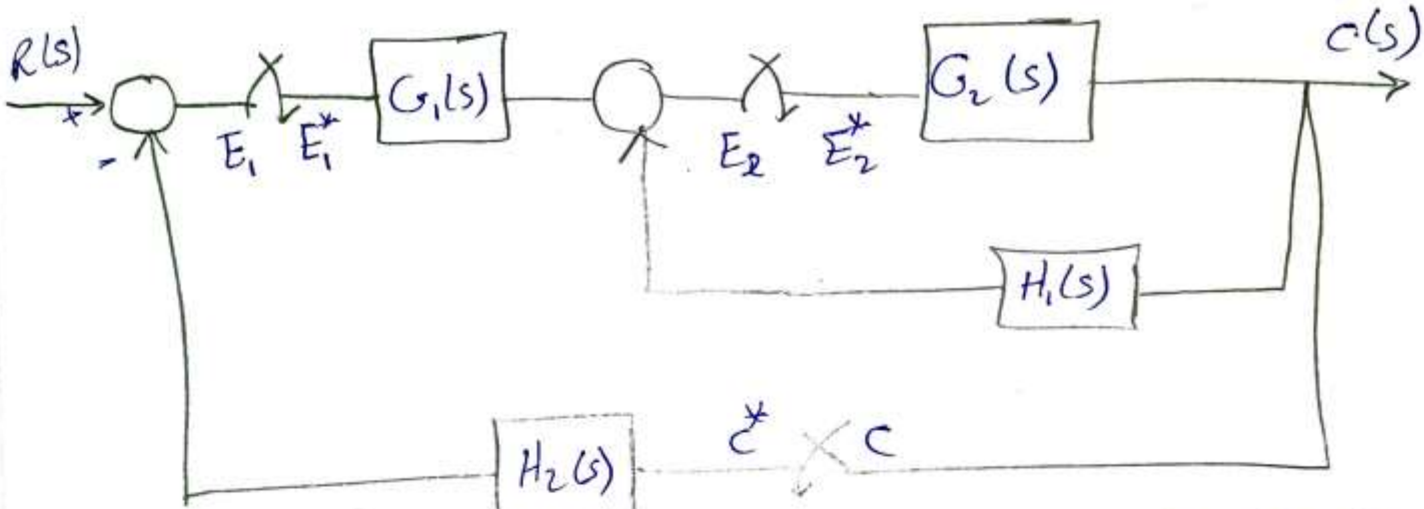
⇐ اذا ~~...~~ قمت بعمل (staring) قبل التعويض مش هتعرف تحسب (Pulse T.F) لكن فيه حالات بنعوض فيها بعد ال (staring ~~...~~

← لو مفيش (Sampler) بين $R(s)$ وأول بلوك يصعب فصلهم

$$\overset{*}{C}(s) = \frac{G_2^* \cdot \overline{G_1 R(s)}^*}{1 + \overline{G_1 G_2 H(s)}^*}$$



← ممكن تحسب (Pulse T.F) بواسطة النظر :-



2 blocks in system.

$$\frac{C(z)}{R(z)} = \frac{G_1(z)\cdot G_2(z)}{1 + \underbrace{\overline{G_2 H_1}(z)}_{\text{(feedback) أول}} + G_1(z)\cdot G_2(z)\cdot H_2(z)}$$

على عين الرسم.

steady state error

$$K_p = \lim_{z \to 1} \overline{GH}(z) \Rightarrow s.s.e = \frac{1}{1+K_p}$$

→ unit step $\Rightarrow r(t) = 1$

$$R(z) = \frac{z}{z-1}$$

For unit ramp

$r(t) = t \longrightarrow r(KT) = KT \Rightarrow R(z) = \frac{TZ}{(Z-1)^2}$

$K_v = \frac{1}{T} \lim_{Z \to 1} (Z-1)\, \overline{GH(z)}$

$s.s.e = \frac{1}{K_v}$

For acceleration i/p $r(t) = \frac{t^2}{2}$

$K_a = \frac{1}{T^2} \lim (Z-1)^2\, \overline{GH(z)}$

$s.s.e = \frac{1}{K_a}$

s.s.e depends on

1) input

2) type of input.

system type and order

for example

$\overline{GH(Z)} = \frac{5}{(Z-1)^3 (Z^2 - 1.2Z + 4)}$

System order $= \cancel{\ } 5$

أكبر أس في المقام

$\rightarrow$ system type $= 3$ $(Z-1)$ الأس بتاع الصفر

ـ الحاجات اللى بتنطلب فى المسائل :-

1) open loop pulse T.F

$$= \overline{GH(z)}$$

2) Closed loop Pulse T.F

$$= \frac{G(z)}{1 + \overline{GH(z)}}$$

3) steady state value of the o/p for unit step input

$$c(\infty) = \lim_{z \to 1} (z-1)\, c(z)$$

<u>or</u> $e(t) = r(t) - y(t)$

$$\underbrace{e(\infty)}_{\text{s.s.e}} = r(\infty) - y(\infty) \longrightarrow \text{steady state o/p}$$

4) First 3 ~~te~~ terms of o/p sequence for unit step, $T = 0.4$

$y(0)$, $y(0.4)$, $y(0.8)$

$y(KT)\big|_{K=0}$ $\quad y(KT)\big|_{K=1}$ $\quad y(KT)\big|_{K=2}$

← إزاي تحسب الثلاثة نقط

$$C(z) = \underbrace{(T.F)}_{\text{closed loop}} \cdot R(z)$$

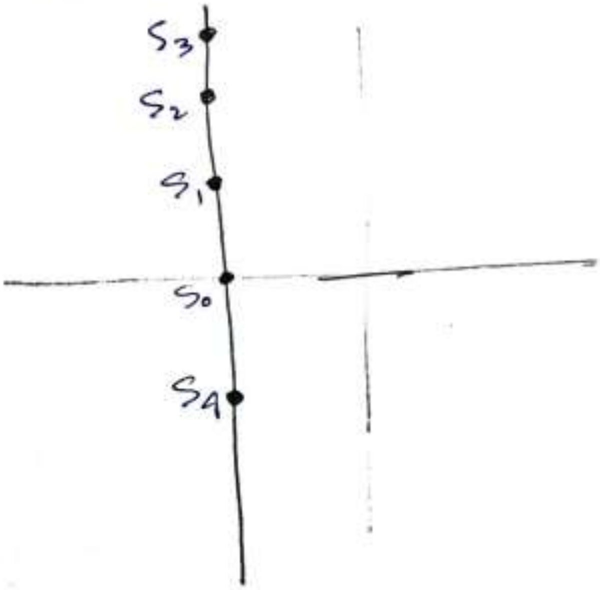
← ثم هيطلع في الحد الأخير كسر تعمل قسمة مطولة وتاخد أول ثلاثة حدود.

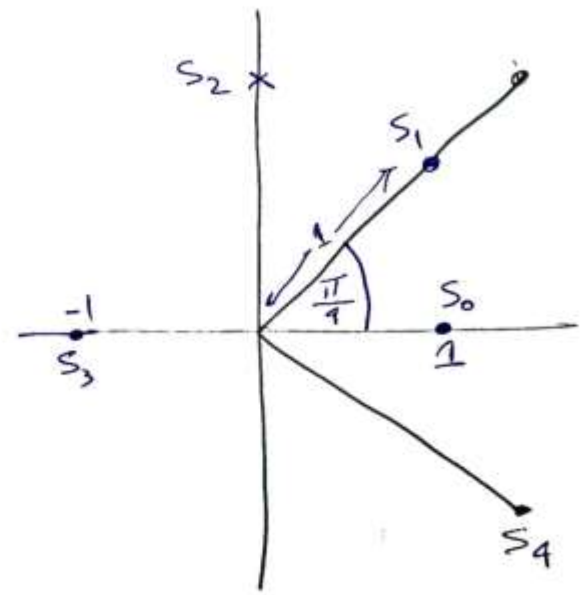
$$ex \rightarrow \underbrace{3}_{c(0)} + \underbrace{1.2}_{c(0.4)} z^{-1} + \underbrace{1.5}_{c(0.8)} z^{-2}$$

Maping from z-domain and s-domain

$$s = \sigma + j\omega \rightarrow z = e^{Ts}$$

$$z = e^{\sigma T} \angle \omega T \Rightarrow \theta = \omega \quad , \quad r = e^{\sigma}$$





$S_0 = 0 + J0 \Rightarrow Z = 1 \angle 0$

$S_1 = 0 + J\frac{\pi}{4} \Rightarrow Z_1 = e^0 \angle \frac{\pi}{4} = 1 \angle 45^\circ$

$S_2 = 0 + J\frac{\pi}{2} \rightarrow Z_2 = 1 \angle \frac{\pi}{2}$

$S_3 = J\pi \longrightarrow Z_3 = 1 \angle \pi$

– لما نعمل (mapping) لـ (imag. axis) في الـ (s-plane)
إلى (z-plane) ينتج دائرة نصف قطرها 1 ومركزها
نقطة الأصل.

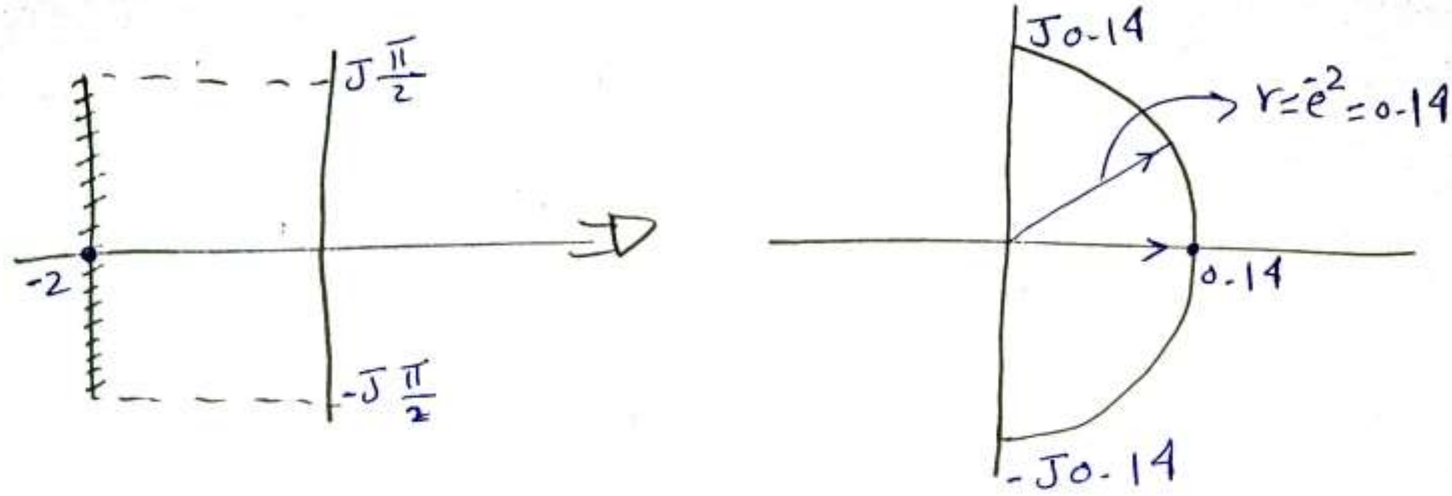
– يتم حساب الـ (stability) طبقاً للدائرة.

$r < 1 \rightarrow$ stable.

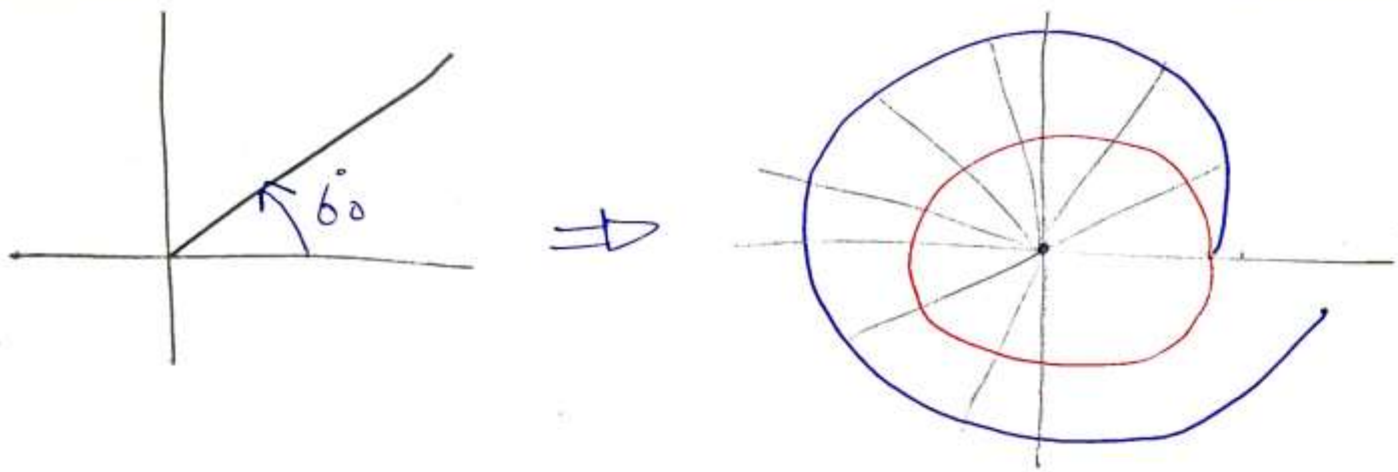
$r > 1 \rightarrow$ unstable.

$r = 1 \rightarrow$ critically stable

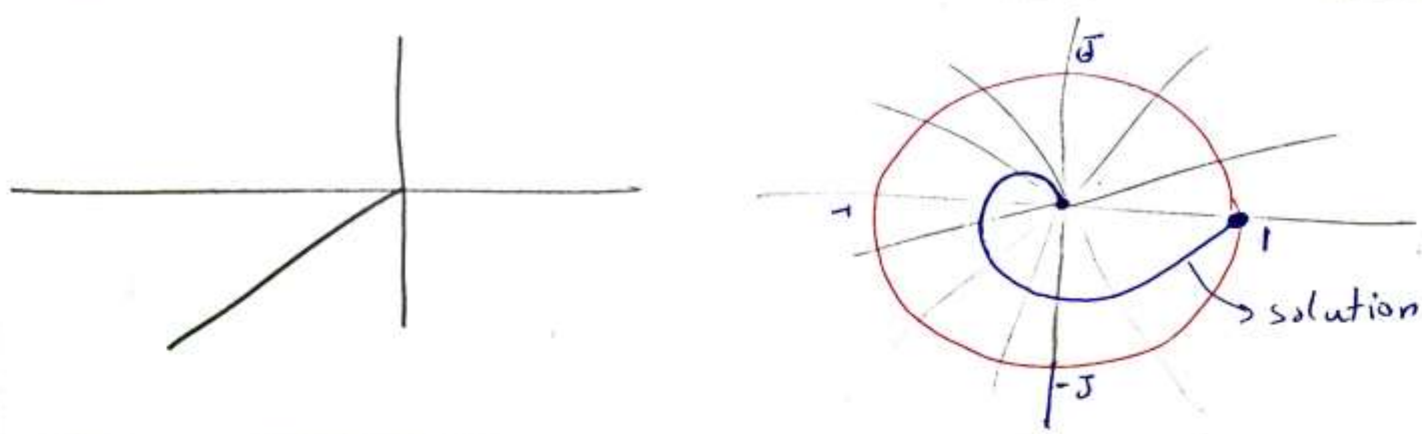
* system is stable if: all poles lies inside unit circle ($|Z_p| = 1$)
* system is unstable if, at least one pole lies outside unit circle.
* system is critical stable, if one or more poles lies on unit circle and other poles lies inside unit circle.



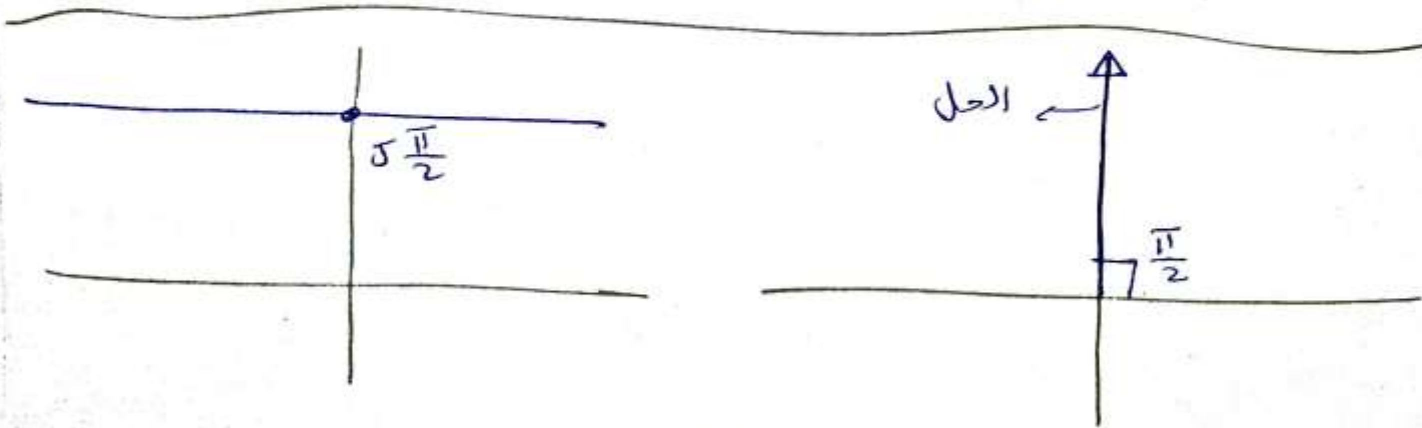
J π/2
-2
-J π/2
J0.14
r = e^-2 = 0.14
0.14
-J0.14



60°



J
-1
1
-J
solution



J π/2
الحل
π/2

check system stability

1) Poles locations

— يعطي معادلة هتفكها وتجيب قيم Z اللي تحدد

if $|z| = 0.7 < 1 \Rightarrow$ stable

$|z| = 0.5, 0.3, 1 \Rightarrow$ critically stable.

$|z| = 0.5, 0.7, 1.5 \Rightarrow$ unstable.

2) using bilinear transformation

— عندك معادلة في Z بتحولها بالقانون :-

$$Z = \frac{1+r}{1-r}$$

— وتفك المعادلة الناتجة حتى تصل لمعادلة في r تحلها

ب (Routh array)

ex $r^3 + 9r^2 - 9r - 81 = 0$

$$\begin{array}{c|cc}
r^3 & 1 & -9 \\
r^2 & 9 & -81 \\
A(r)\quad r^1 & 0 & \\
\hline
r^1 & 18 & \\
r^0 & -81 &
\end{array}$$

$A(r) = 9r^2 - 81$

$\frac{dA(r)}{dr} = 18r$

— الإشارة تغيرت يكون

system unstable.

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3) using Jury test

$F(z) = 1 + \cancel{\ } \overline{GH(z)}$

$F(z) = a_n z^n + a_{n-1} z^{n-1} + \dots\dots + a_0$

system to be stable

1) $F(1) > 0$; 2) $(-1)^n F(-1) > 0$

3) $|a_0| < |a_n|$ $n \rightarrow$ system order

ـــ الثلاث نقط دي نستخدم في حالة (2nd order sys.)

ـــ اكبر منه كده نستخدم (Jury matrix)

	z^0	z^1	z^2	....	z^{n-1}	z^n
1	a_0	a_1	a_2	----	a_{n-1}	a_n
2	a_n	a_{n-1}	a_{n-2}	----	a_1	a_0
3	b_0	b_1	----		b_{n-1}	
4	b_{n-1}	b_{n-2}	----		b_0	
2n-3	r_0	r_1	r_2			

4) $|b_0| > |b_{n-1}|$

5) $|r_0| > |r_2|$

← في المصفوفة السابقة كي تحسب قيم b ، a

$$b_0 = \begin{vmatrix} a_0 & a_n \\ a_n & a_0 \end{vmatrix} \quad ; \quad b_1 = \begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix}$$

← لو اول شرطين طلعوا :

$F(1) = 0$ or $(-1)^n F(-1) = 0$

← هتكمل حل باقي الشروط لو الباقي تحقق يكون (system critically stable).

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if system order

1) equal to 2 → use Jury test

2) greater than 2 → you better use Routh array.

In Jury when you check stability

← نستخدم اول ثلاثة شروط فقط كل شرط يطلع معادله لـ K تقاطع هو $\begin{pmatrix}\text{range of } K \\ \text{for stability}\end{pmatrix}$

stability

*Relative stability

→ to what range system is stable.

using GM & PM

ways:

- → Bode diagram
- → Polar Plot
- → Nyquist

*absolute stability

→ Tell us the stability of system (stable, unstable, critically stable)

- → Jury
- → bilinear Transformation (Routh)

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stability

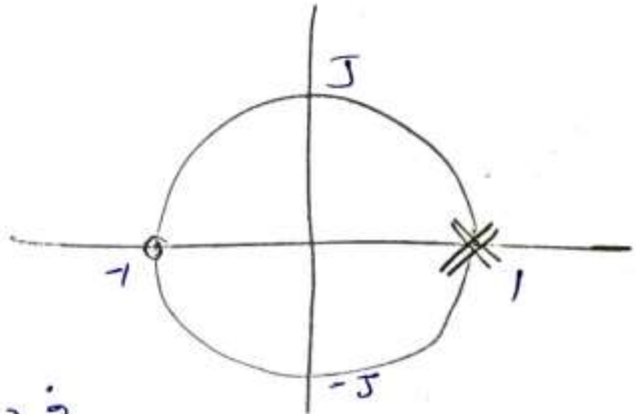
1) Graphical methods

1) Root locus
2) Bode diagram
3) Polar Plot
4) Nyquist

2) Algebric methods

1) Jury test
2) Routh array (using bilinear transformation)

ـــ طريقة حل مسائل ال (root locus)



1) Real Part

بتقف ~~وبرد~~ عند كل (Pole) or Zero
~~وتصعد للشمال~~
وتنظر للشمال لو عينك عدد فردى من (Poles, zeros) تكون real Part

في الرسمة ←

Real Part ⇒ from -1 to -∞

قبل نقطة (1) فيه ال Poles ، zeros وموقعهم في ال (Z-Plane)

2) Asymptotes

a) no. of Asym. $= n_p - n_z$

b) Center of Asym. $(C_A) = \dfrac{\sum Poles - \sum zeros}{n_p - n_z}$

c) $\Theta = \dfrac{(2L+1)180}{n_p - n_z}$

3) Breaking Point

Ch. equation $1 + \overline{GH(z)} = 0$

ـــ ينتج معادلة لـ K نحسب تفاضلها $\dfrac{dK}{dz} = 0$

ـــ تنتج قيم ال (Breaking)

Breakaway ، Break in.

← شكل المسائل :

١) يجيب ليك (block) فيه (ZOH)

1) find open loop T.F $\overline{GH(z)}$

2) it will get a function, then continue by steps in previous page.

<u>if</u> $\overline{GH(z)} = \frac{K}{4} * \frac{5z+1}{z(z-1)}$

put $\frac{K}{4} = \grave{K}$

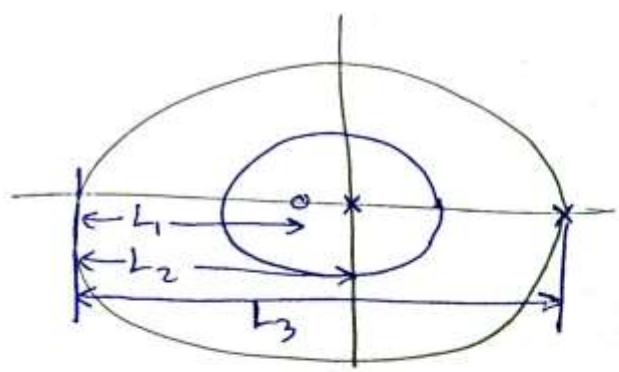
$\hookrightarrow GH(z) = \frac{\grave{K}(5z+1)}{z(z-1)}$

2) يلخص عليك ويعطي $\overline{GH(z)}$ مباشرة.

* حساب ال K_{cr}

<u>1) from this law</u>

$$K = \frac{\Pi \text{ poles}}{\Pi \text{ zeros}}$$

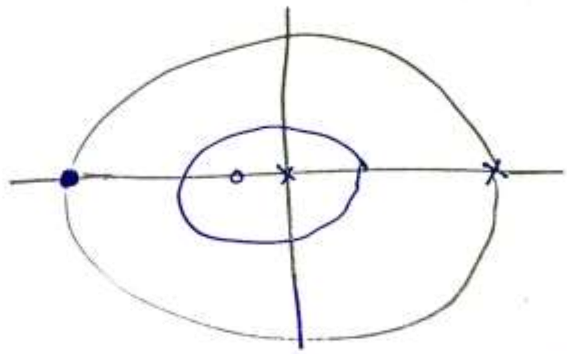


<u>في الرسم</u>

$$K = \frac{L_2 L_3}{L_1}$$

2) another solution

ـ إتباع الخطوات حسب قيمة K وطلعت معادلة معينة هتستخدمها تاني وتعوض $Z=-1$.



$$K = -\left[\frac{Z(Z-1)}{Z+0.2}\right]$$

Put $Z=-1$

$K = vv$

3) using Jury test $(1+\overline{GH(z)}=0)$

ـ نستخدم أول ثلاثة حالات فقط.

4) using bilinear transformation.

ـ نستخدم المعادلة $1+\overline{GH(z)}$ والتانوم

$$Z = \frac{1+r}{1-r}$$

→ then you use routh array

→ you will face some functions.

→ interaction of them is range of K.

→ some times you may substitute functions.

→ to study system Properties in freq. domain for a discrete time system, we use bilinear transformation, to get system in Continuous time domain. $Z = \frac{1+r}{1-r}$

Bode Diagram

→ relative stability method.

⇒ To draw Bode diagram:-

1) Map from Z-domain to r-domain.

$$Z = \frac{1+r}{1-r} \Rightarrow \overline{GH(z)} \longrightarrow \overline{GH(r)}$$

2) replace $r \longrightarrow j\omega_r$

$$\overline{GH(r)} \longrightarrow \overline{GH(j\omega_r)}$$

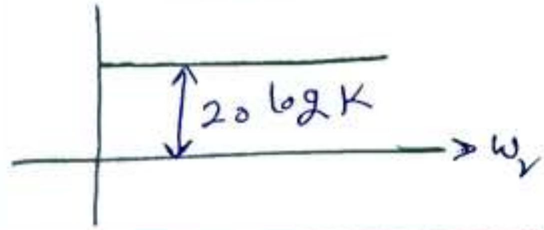
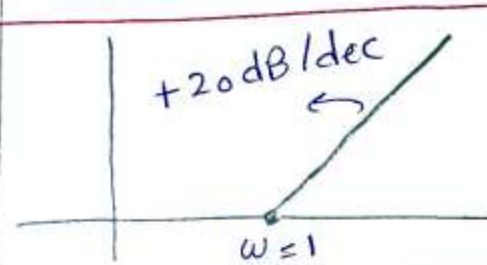
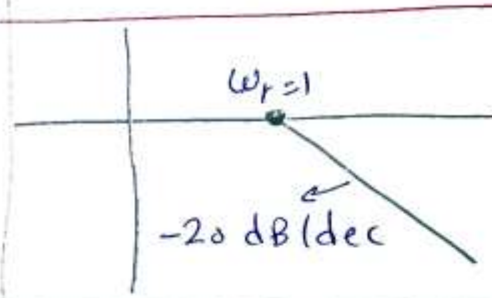
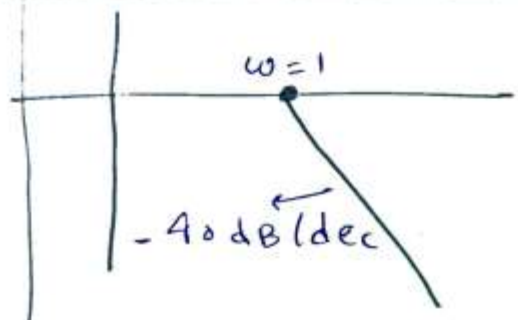
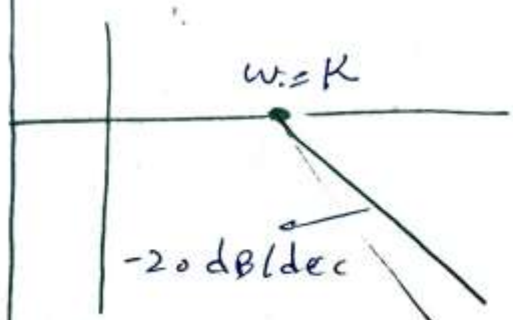
3) $\left|\overline{GH(j\omega_r)}\right| = \frac{|\text{بسط}|}{|\text{مقام}|}$

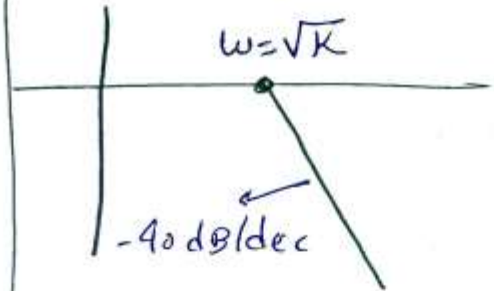
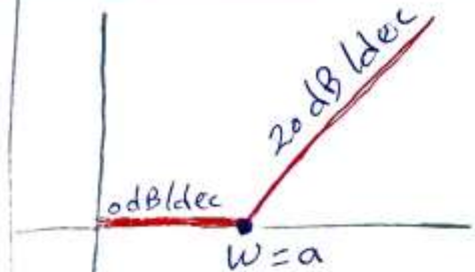
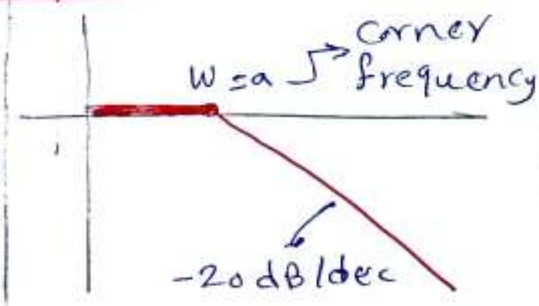
$$\left|\overline{GH(j\omega_r)}\right|_{dB} = 20 \log \left|\overline{GH(j\omega_r)}\right|$$

4) $\phi(\omega_r) = \angle \text{بسط} - \angle \text{مقام}$

$$= \tan^{-1}\left(\frac{\text{Imaginary}}{\text{real}}\right)$$

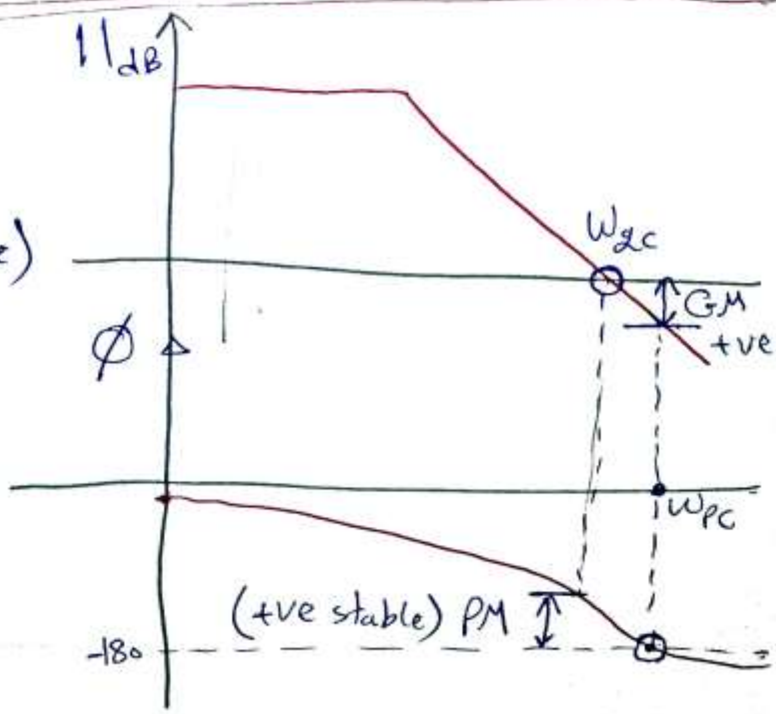
Common terms

Term	$\phi(\omega_r)$	$\| \|_{dB}$
K	—	
$s \to r \to j\omega_r$	$+90$	
$\frac{1}{s} \to \frac{1}{r} \to \frac{1}{j\omega_r}$	-90	
$\frac{1}{s^2} \to \frac{1}{r^2} \to \frac{1}{j\omega_r} \cdot \frac{1}{j\omega_r}$	-180	
$\frac{K}{s} \to \frac{K}{r} \to \frac{K}{j\omega_r}$	-90	

<table>
<tr><th>Term</th><th>$\phi(\omega_r)$</th><th>$| \; |_{dB}$</th></tr>
<tr><td>$\frac{K}{s^2} \longrightarrow \frac{K}{r^2}$</td><td>$-180$</td><td></td></tr>
<tr><td>$\left(1+\frac{s}{a}\right) \to 1+\frac{r}{a}$
$\hookrightarrow 1+J\frac{\omega_r}{a}$</td><td>$\tan^{-1}\left(\frac{\omega_r}{a}\right)$</td><td></td></tr>
<tr><td>$\frac{1}{1+\frac{s}{a}} \to \frac{1}{1+\frac{r}{a}}$
$\to \frac{1}{1+J\frac{\omega_r}{a}}$</td><td>$-\tan^{-1}\left(\frac{\omega_r}{a}\right)$</td><td></td></tr>
</table>

GM
- > 0 (stable)
- < 0 (unstable)
- $= 0$ (critically stable)

← لو عندك قيمتين
لل (GM) تأخذ أصغرهم.



ـــ يوجد مثال على الـ (Bode diagram) آخر محاضرة رقم ٦ .

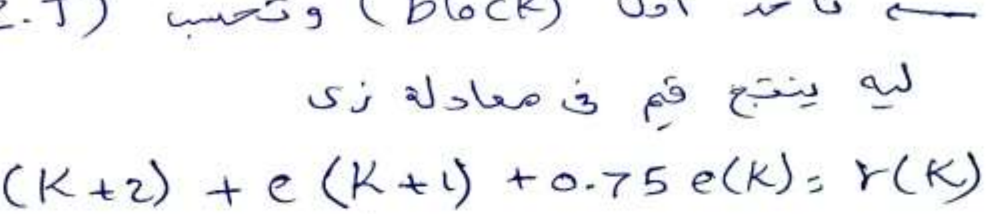
: state -space model

* representation of state variable model:-

1) Controller Canonical form.
2) observer " " .
3) Parallel / diagonal form.
4) Cascaded form:

$\boxed{Ex}$ $$\frac{Y(z)}{R(z)} = \frac{a_1 z^2 + a_2 z + a_3}{z^3 + b_2 z^2 + b_3 z + b_4}$$

$\boxed{1}$ for Controller form.



ـــ تأخذ أول (block) وتحسب (Inverse Z.T) ليه ينتج قيم في معادلة زي

$$e(K+3) + e(K+2) + e(K+1) + 0.75\, e(K) = r(K)$$

Put: $e(K) = X_1 \longrightarrow e(K+1) = X_1(K+1)$

$e(K+1) = X_2 \longrightarrow e(K+2) = X_2(K+1)$

$e(K+2) = X_3 \longrightarrow e(K+3) = X_3(K+1)$

ـــ تنتج معادلة تقدر تحل منها ال $X(K+1)$

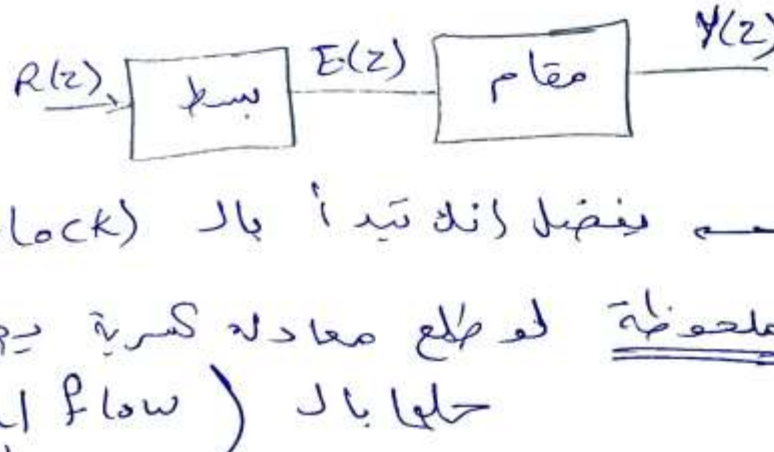
$$X(K+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -b_4 & -b_3 & -b_2 \end{bmatrix} X(K) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(K)$$

ـــ بعدين تيجي لل (block) التاني وتستخدم تعويضات ال (block) الأول وتحسب $y(K)$

$$y(K) = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} X(K)$$

2) observer canonical form

ـــ هتمشي عكس ال (Controller)



ـــ يفضل انك تبدأ بال (block) التاني اللي فيه المقام.

ملحوظة لو طلع معادلة كسرية يصعب التعامل معها حلها بال (signal flow graph) حتى تجد $X(K+1)$ ~~...~~

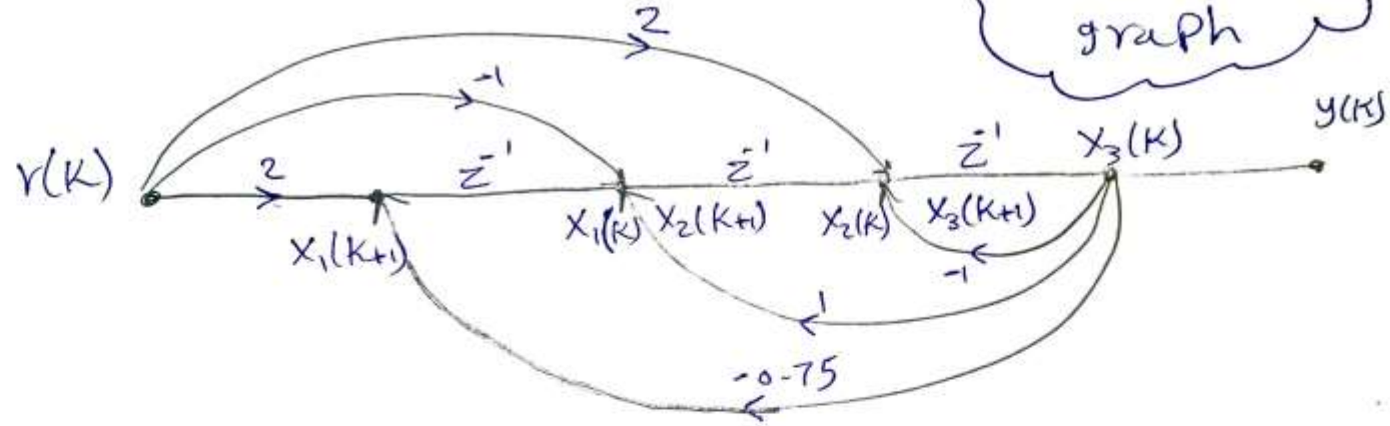
له المثال في محاضرة رقم ٧

for ex

لو الأس موجبة
اضرب بسطا ومقاما
في أكبر أس لكن
بالسالب

$$\frac{Y(z)}{R(z)} = \frac{2z^{-1} - z^{-2} + 2z^{-3}}{1 - \left[-z^{-1} + z^{-2} - 0.75\, z^{-3}\right]}$$

signal flow graph



$$X_1(K+1) = -0.75\, X_3(K) + 2\, r(K)$$

$$X_2(K+1) = X_1(K) - r(K) + X_3(K)$$

$$X_3(K+1) = X_2(K) + 2r(K) - X_3(K)$$

$$y(K) = X_3(K)$$

يكون الشكل في ال (observer) كالآتي

$$X(K+1) = \begin{bmatrix} 0 & 0 & -b_4 \\ 1 & 0 & -b_3 \\ 0 & 1 & -b_2 \end{bmatrix} X(K) + \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} r(K)$$

$$y(K) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X(K)$$

← ممكن نستخدم الـ (block diagram)
لحل المسائل برده بدل الـ $\begin{pmatrix}\text{signal flow}\\ \text{graph}\end{pmatrix}$
لو موجود دي محاضرة ٧ لمن يريد استخدامها.

$\boxed{3}$ diagonal form

$$\frac{Y(z)}{R(z)} = \frac{A}{Z-b_1} + \frac{B}{Z-b_2} + \frac{C}{Z-b_3}$$

← لو المسألة غير كده بتوصلها للشكل ده باستخدام
(Partial fraction)

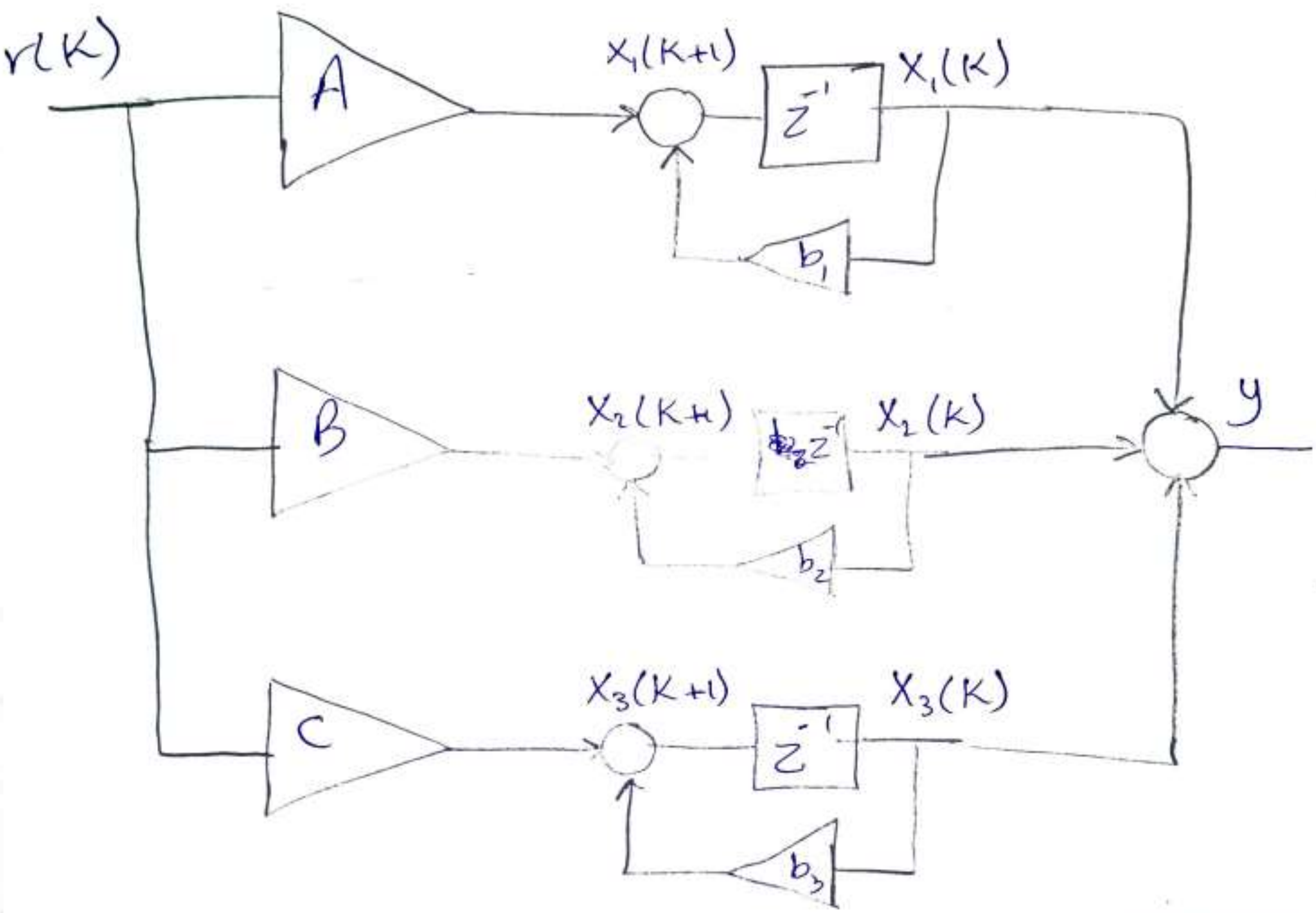
$$Y(z) = \underbrace{\frac{A}{Z-b_1}R(z)}_{\to X_1(z)} + \underbrace{\frac{B}{Z-b_2}R(z)}_{\to X_2(z)} + \underbrace{\frac{C}{Z-b_3}R(z)}_{\to = X_3(z)}$$

← هتحسب (inverse Z.T) لكل منهم لوحده.

$$x(K+1) = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix} x(K) + \begin{bmatrix} A \\ B \\ C \end{bmatrix} r(K)$$

$$y(K) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x(K)$$

← في ال (diagonal form) الرسم مش مطالب بيه
لكنه ممكن يطلبه أحيانًا في السؤال.



state-space analysis

$$x(K+1) = A\,x(K) + B\,u(K)$$

$$y(K) = C\,x(K) + D\,u(K)$$

$$\underline{\underline{T.F}} = \frac{Y(z)}{U(z)} = C(zI-A)^{-1}B + D$$

2) Ch. equation

$$|zI - A| = 0$$

3) Controllability:-

→ system is controllable if for any change of an external input, produce change in internal states of system.

~~4) ob~~ → controllability matrix (M_c)

$$M_c = (B \quad AB \quad A^2B \cdots A^{n-1}B)$$

if $|M_c| \neq 0$ → system is controllable.

4) observability

→ system is observable, if we can estimate the states values from relation between input and output or by history information from the o/p and i/p.

$$M_o = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{pmatrix}$$

if $|M_o| \neq 0$

↳ system is observable.

4 system response:-

Let $\phi(z) = (zI - A)^{-1}$

$$x(z) = \phi(z) \cdot z \cdot x(0) + \phi(z) \cdot B \cdot u(z) \quad \Big\downarrow z^{-1} \cdot T$$

$x(K) = \checkmark\checkmark$

then: system response $y(K)$

$$y(K) = C\,x(K) + D\,u(K)$$

لا إثبات موجود في
أول محاضرة ٩

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + D\,u(t)$$

Discretization $\downarrow$ (Sampler + Z.O.H)

$$x(K+1)T = A_d\,x(KT) + B_d\,u(KT)$$
$$y(KT) = C_d\,x(KT) + D_d\,u(KT)$$

$A_d = \phi(T)$, $C_d = C$, $D_d = D$

$$B_d = \int_0^T \phi(q) \cdot B \cdot dq$$

Where:

$$\phi(T) = \phi(t)\Big|_{t=T}$$

$$\phi(t) = \mathcal{L}^{-1}(SI-A)$$

$$\phi(q) = \phi(t)\Big|_{t=q}$$

Control design

Classical

PI, PD, PID

Phase lead, Phase lag...

Modern

- state feedback Control
- observer design

Control Problem

regulation Problem ($r=0$)

- Concerned with disturbance or noise rejection
- there is no input.
- example: state feed back control.

servo Problem ($r \neq 0$)

- Concerned with enhancement of system Performance in Presence of desired reference input.
- input existed.

→ state feeback control

$$u(KT) = -K\,x(KT)$$

⇒ we want to find gain matrix K

$$K = [K_1 \quad K_2 \;\dots\; K_n] \quad n \rightarrow \text{system order.}$$

$\boxed{\text{first method}}$

a) desired ch. equation $\alpha_c(z)$

$$\alpha_c(z) = (z - z_1)(z - z_2)\dots\dots \rightarrow ①$$

b) using $u(KT) = -K\,x(KT)$

$$x(K+1)T = A_d\,x(KT) + B_d\,\underbrace{u(KT)}_{= -K\,x(KT)}$$

ch. eq. $= |zI - A_d + B_d K| = 0 \rightarrow ②$

→ Compare ① , ② to get K.

$\boxed{\text{2nd method : Ackerman method}}$

$$K = [K_1 \quad K_2 \dots K_n] = (0 \;\; 0 \dots 1)\,M_c^{-1}\,\alpha_c(A)$$

$$M_c \rightarrow (B \quad AB \;\dots\dots\; A^{n-1}B)$$

$$\alpha_c(A) = \alpha_c(z)\Big|_{z=A}$$

ـ هنا بتبدأ بعد نقطة
a في الطريقة الأولى.

Notes

← ممكن يجيلك ال (Poles) على الشكل الآتي :

ζ , ω_n

$$S_{1,2} = -\zeta \omega_n \pm J \omega_n \sqrt{1-\zeta^2}$$

$$r = e^{-\zeta \omega_n T} \quad \& \quad \theta = \omega_n T \sqrt{1-\zeta^2}$$

$$Z_{1,2} = r\cos\theta \pm J r \sin\theta$$

→ How to get M_c^{-1}

assume $M_c = \begin{pmatrix} 2 & -6 \\ 2 & 5 \end{pmatrix}$

$$M_c^{-1} = \frac{1}{-(2*6)+(2*5)} \begin{pmatrix} 5 & 6 \\ -2 & 2 \end{pmatrix}$$

---

observer design

← يجب أن تكون جميع ال (states) قابلة للقياس

ولو حصل وتواجد قيمة أو قيم غير قابلة للقياس يتم

تصميم (observer) لحساب ال (states) دي.

~~حساب~~ الحساب هنا هيكون من قيم الدخل

والخرج السابقة.

observer eqn

$$\hat{x}(K+1) = (A - GC)\hat{x}(K) + Bu(K) + Gy(K)$$

$$G \rightarrow (\text{gain matrix}) = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix}$$

$\rightarrow$ gain matrix determined through specs of observer as:

* observer speed response
* transient.
* settling time

} ζ, ω_n منها تحسب
ومنها (desired Poles)

To determine gain matrix

first method

a) desired ch. equation $\alpha_o(z)$

$$\alpha_o(z) = (z - P_1)(z - P_2) \cdots (z - P_n) = 0 \rightarrow (1)$$

b) obeserver ch. equation:-

$$|zI - A + GC| = 0 \longrightarrow (2)$$

Compare (1), (2) to get G

2 using Ackermann's method

← بعد حساب (desired ch. equation)

$$G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} = \alpha_o(A) \cdot M_o^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

$$\alpha_o(A) = \alpha_o(z)\Big|_{z=A}$$

← ممكن في سؤال يعطيك ζ ، وقيمة T_c في ال (Controllable) وعلاقة تربطها بـ $T_{observer}$ ومعطى عندك قيمة $Z_{1,2}$

ex $\boxed{\tau_o = \frac{1}{2}\tau_c}$

for example

$Z_{1,2} = 3 \pm j2.5$

$\hookrightarrow r = \sqrt{3^2 + 2.5^2}$

$\hookrightarrow r = e^{\frac{-T}{\tau_c}}$

$\ln(r) = \frac{-T}{\tau_c}$

$\rightarrow$ find τ_c then find τ_o

$\rightarrow \tau_o = \frac{1}{\omega_n \zeta} \Rightarrow \omega_n = ..$

← وكمل الحل عادي.